Using Symmetries to Investigate Complete Integrability of Nonlinear PDEs and Differential-Difference Equations

Willy Hereman
Department of Applied Mathematics and Statistics
Colorado School of Mines, Golden, CO

Abstract

Originated by Sophus Lie, continuous transformation groups are frequently used to compute analytic solutions of ordinary and partial differential equations. Lie point symmetries (which include discrete symmetries as a subset) are also useful to investigate the complete integrability of nonlinear PDEs and differential-difference equations (DDEs).

As an example, it will be shown how scaling symmetries can be used in the algorithmic computation of conservation laws, higher-order symmetries, recursion operators, and Lax pairs of nonlinear polynomial PDEs and DDEs.

The approach will be illustrated with well-known PDEs from soliton theory, including the Korteweg-de Vries (KdV) equation and its generalizations as well as the modified KdV and Boussinesq equations.

Drawing on the similarities with PDEs, the same approach can be used to study the complete integrability of nonlinear DDEs, for example, the Kac-van Moerbeke, Volterra, and Toda lattices.