Collapse Versus Blow-Up and Global Existence in the Generalized Constantin-Lax-Majda Equation with dissipation

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Abstract

We analyze the dynamics of singularities and finite time blowup of generalized Constantin-Lax-Majda equation which corresponds to non-potential effective motion of fluid with competing convection and vorticity stretching terms. Both non-viscous fluid and fluid with various types of dissipation including usual viscosity are considered. An infinite families of exact solutions are found together with the different types of complex singularities approaching the real line in finite times. Both solutions on the real line and periodic solutions are considered. In the periodic geometry, a global-in-time existence of solutions is proven when the data is small and dissipation is strong enough. The found analytical solutions on the real line allow finite-time singularity formation for arbitrarily small data, even for various form of dissipation, thereby illustrating a critical difference between the problems on the real line and the circle. The analysis is complemented by accurate numerical simulations, which are able to track the formation and motion singularities in the complex plane. The computations validate and extend the analytical theory.

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