Solitons and soliton interactions in the complex coupled short-pulse equation

Aikaterini Gkogkou

Department of Mathematics, University at Buffalo

Abstract

Ultra short-pulses are pulses whose width is of the order of 10^{-15} seconds and much smaller than the carrier frequency. The understanding of their propagation is very important because they allow very high data transmission in one channel. The complex coupled short-pulse equation (ccSPE) describes the propagation of ultra-short optical pulses in nonlinear birefringent fibers. The system admits a variety of vector soliton solutions: fundamental solitons, fundamental breathers, composite breathers, and self-symmetric solitons which are special cases of composite breathers. In this talk, we discuss the nature of ccSPE soliton interactions. We use Manakov's method to describe the interaction between two fundamental solitons, in which case there exists redistribution of energy between the components, unless the initial polarization vectors of the solitons are either parallel or orthogonal. Manakov's method can also be modified to describe the interaction between two self-symmetric solitons and the interaction between a fundamental soliton and a self-symmetric soliton, but it is no longer effective to describe more complicated soliton interactions. In this case, we rely on Darboux matrices corresponding to the various types of solitons, combining refactorization problems on generators of certain rational loop groups and long-time asymptotics of these generators. This leads to the derivation of various Yang-Baxter maps for the polarizations of the solitons, which allows to completely characterize all types of soliton interactions. Specifically, we reproduced the results we derived with Manakov's method, but we were also able to investigate more complicated soliton interactions: when a fundamental

soliton interacts with a fundamental breather always turns into a fundamental breather, while the interaction of two fundamental breathers generically yields two fundamental breathers but may also result into a fundamental soliton and a fundamental breather.

References

[1] Manakov S.V., “On the theory of two-dimensional stationary self-focusing of electromagnetic waves”. Sov. Phys. JETP 38, 248–253 (1974).

[2] A. Boutet deMonvel, D. Shepelsky, L. Zielinski, “The short pulse equation by a Riemann–Hilbert approach”. Lett. Math. Phys., 107 (2016), 1345–1373.

[3] B. Prinari, A.D. Trubatch, B-F Feng, “Inverse scattering transform for the complex short-pulse equation by a Riemann–Hilbert approach”. Eur. Phys. J. Plus, 135 (2020), 717–735.

[4] A. Gkogkou, B. Prinari, A.D. Trubatch, B-F Feng, “Inverse scattering transform for the complex coupled short-pulse equation by a Riemann-Hilbert approach”. Stud. App. Math., 148 (2022), 918–963.

[5] V. Caudrelier, A. Gkogkou, B. Prinari, “Soliton interactions and Yang-Baxter maps for the complex coupled short-pulse equation”. Stud. App. Math., submitted (2023).