A simplified Hirota method: Computation of solitary wave solutions and solitons through homogenization of degree

Willy Hereman

Department of Applied Mathematics and Statistics, Colorado School of Mines

Abstract

Hirota's method is an effective method to find soliton solutions of completely integrable nonlinear PDEs, including the KdV, mKdV, NLS, and sine-Gordon equations. Hirota's approach requires a change of the dependent variable (a.k.a. Hirota's transformation) so that the resulting equation can be written in bilinear form using the Hirota operators. Solitons are then computed using a perturbation scheme that terminates after a finite number of steps.

It will be shown that the Hirota transformations are crucial to obtain PDEs that are homogeneous of degree (in the new dependent variables). The actual recasting into bilinear form which assumes a quadratic equation (or a tricky decoupling into such equations) is not required to compute solitary wave solutions or solitons. To illustrate this idea, soliton solutions of a class of fifth-order KdV equations (due to Lax, Sawada-Kotera, and Kaup-Kupershmidt) will be computed with a straightforward recursive algorithm involving linear and nonlinear operators. Although it circumvents bilinear forms, this method can still be viewed as a simplified version of Hirota's method.

Homogenization of degree also allows one to find solitary wave solutions of nonlinear PDEs that are either not completely integrable or for which the bilinear form is unknown. A couple of such examples will also be shown.