Finite-time singularity formation in the generalized Constantin-Lax-Majda equation

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The question of finite-time singularity formation for solutions to the generalized Constantin-Lax-Majda (gCLM) equation is considered. This equation was first introduced by Constantin, Lax and Majda as a simplified model for singularity formation in the 3D incompressible Euler equations. It was later generalized by Okamoto, Sakajo and Wensch to include an advection term with parameter a, which allows different relative weights for advection and vortex stretching. There has been intense interest in the gCLM equation, and it has served as a proving ground for the development of methods to study singularity formation in the 3D Euler equations. Despite significant effort, little is known about singularity formation in the gCLM equation for general values of a. In this talk we provide such information via a combination of analysis and numerical computations for both the inviscid and the dissipative (or viscous) versions of the equation. Our findings include the existence of a critical value of a at which the nature of singularity formation drastically changes type. We also find a significant difference between the problems in the periodic and real-line geometries when dissipation is present.