

Fibonacci Turbulence

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Abstract

We introduce a simple yet rich family of discrete models from the class of shell models for turbulence. The model is Hamiltonian, with triplet interactions of neighboring modes; it has quadratic conservation laws defined by the Fibonacci numbers. Depending on how the interaction time changes with the mode number, three types of turbulence are found: a single direct cascade, a single inverse cascade (without an accompanying direct cascade for another conserved quantity), and a double cascade, where direct and inverse cascades exist simultaneously. The latter case allows us to study turbulence close to thermal equilibrium, which happens when the typical interaction time weakly depends on the mode number. In this case, when the length of the cascade increases, the one-mode statistics approaches the equilibrium Gaussian distribution, and the dimensionless three-mode cumulant describing the energy flux is small. Yet, surprisingly, we find that higher multi-mode cumulants do not decrease but actually grow with the number of modes involved. Our findings demonstrate that the multi-mode correlations is the most sensitive and sensible way to characterize the radical difference between turbulence and thermal equilibrium.