

Weighted Birkhoff Averages as an Efficient Chaos Discriminant

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Abstract

According to Birkhoff, the time average of a function on phase space should equal the space average when the dynamics are ergodic. However, such time averages may converge very slowly—and indeed the slowest convergence occurs for chaotic orbits. Here we show that the Weighted Birkhoff Average (WBA) method pioneered by Das, Sander and Yorke can be used as an efficient technique to distinguish chaotic and regular orbits in symplectic and volume preserving maps. Indeed regular orbits generically lie on invariant tori that have Diophantine rotation numbers, and for these it can be shown that the WBA is super-convergent. Computing rotation numbers efficiently and accurately allows one to distinguish between resonant “island” orbits and rotational invariant tori. This is joint work with Evelyn Sander of GMU.