Generalized Constantin-Lax-Majda Equation: Collapse vs. Blow Up and Global Existence

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Abstract

We investigate the behavior of the generalized Constantin-Lax-Majda (CLM) equation which is a 1D model for the advection and stretching of vorticity in a 3D incompressible Euler fluid. Similar to Euler equations the vortex stretching term is quadratic in vorticity, and therefore is destabilizing and has the potential to generate singular behavior, while the advection term does not cause any growth of vorticity and provides a stabilizing effect. We study the influence of a parameter a which controls the strength of advection, distinguishing a finite time singularity formation (collapse or blow-up) vs. global existence of solutions. We find a new critical value a_c=0.6890665337007457... below which there is finite time singularity formation that has a form of self-similar collapse, with the spatial extent of blow-up shrinking to zero, and above which up to a=1 we have an expanding blow up solutions. We identify the leading order complex singularity for general values of a which controls the leading order behavior of the collapsing solution. We also re-derive a known exact collapsing solution for a=0 and we find a new exact analytical collapsing solution at a=1/2. For a_c < a ≤1, we find a blow-up solution on the real line in which the spatial extent of the blow-up region expands infinitely fast at the singularity time. For a>1, we find that the solution exists globally with exponential-like growth of the solution amplitude in time.