Series Strategies and Tests for Convergence

Test Name	When to Use	Details
Known Series – Geometric	When you recognize that the terms are a constant raised to some power	$\sum_{n=0}^{\infty} a \cdot r^n \begin{cases} \text{converges to } \dfrac{a}{1-r} \text{ if } \lvert r \rvert < 1 \\ \text{diverges if } \lvert r \rvert > 1 \end{cases}$
Known Series – *p* series	When you recognize that the terms are $\frac{1}{n}$ raised to a constant power.	$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges when } p > 1 \\ \text{diverges when } p \leq 1 \end{cases}$
Limit test ("Bouncer" test)	If you can easily see how the a_n's behave *as a sequence.*	If $\lim_{n\to\infty} a_n \neq 0$, the series diverges. If $\lim_{n\to\infty} a_n = 0$, the test is inconclusive.
Alternating Series Test	When the series is alternating. Usually, you'll see a $(-1)^n$, $(-1)^{n-1}$, or $(-1)^{n+1}$. Sometimes you'll see a $\sin\left(\frac{2n \pm 1}{2}\pi\right)$ or a $\cos(n\pi)$	Given $\sum (-1)^n b_n$: IF: 1. $b_n > 0$ for all n AND 2. $b_{n+1} > b_n$ for all n AND 3. $\lim_{n\to\infty} b_n = 0$ THEN the series converges. IF: 3. fails, then the series diverges. IF: 1. or 2. fail, but 3. Holds, then the test is inconclusive.

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Absolute Convergence	When some terms of the series are positive, and some are negative, but the series is not alternating.	IF $\sum \lvert a_n \rvert$ converges, then $\sum a_n$ converges (and we say that $\sum a_n$ converges absolutely). IF $\sum \lvert a_n \rvert$ diverges, then the test is inconclusive.
Ratio Test	When you see factorials Sometimes, when you see terms to the n^{th} power. Do Not Use for rational or algebraic functions of *n* (the test will always come out inconclusive)	IF $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert \begin{cases} <1, \text{ converges (absolutely)} \\ >1, \text{ diverges} \\ =1, \text{ the test is inconclusive} \end{cases}$
Root Test	When the terms are something raised to the n^{th} power.	IF $\lim_{n\to\infty} \sqrt[n]{a_n} \begin{cases} <1, \text{ converges (absolutely)} \\ >1, \text{ diverges} \\ =1, \text{ the test is inconclusive} \end{cases}$
Comparison Test	Rational or algebraic functions of *n*. Usually compare to a *p*-series	Can only use when the series $\sum a_n$ and $\sum b_n$ have positive terms!! Suppose $a_n \le b_n$ for each *n*. IF $\sum b_n$ converges, then so does $\sum a_n$. IF $\sum a_n$ diverges, then so does $\sum b_n$.

Test Name	When to Use	Details
Limit Comparison Test	Use for comparison when the comparison test is too vexing.	Can only use when the series $\sum a_n$ and $\sum b_n$ have positive terms!! IF $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ AND $L \neq 0$ AND $L \neq \infty$ THEN $\sum a_n$ and $\sum b_n$ behave the same way.
Integral Test	If the integral is easy to evaluate.	IF $\sum_{n=r}^{\infty} a_n = \sum_{n=r}^{\infty} f(n)$ AND $f(x)$ is continuous on $[r, \infty)$ AND $f(x) > 0$ on $[r, \infty)$ AND $f(x)$ is decreasing on $[r, \infty)$, THEN $\sum_{n=r}^{\infty} a_n$ behaves the same way as $\int_r^{\infty} f(x)dx$.