Atomic Decomposition of Some Banach Spaces and Applications

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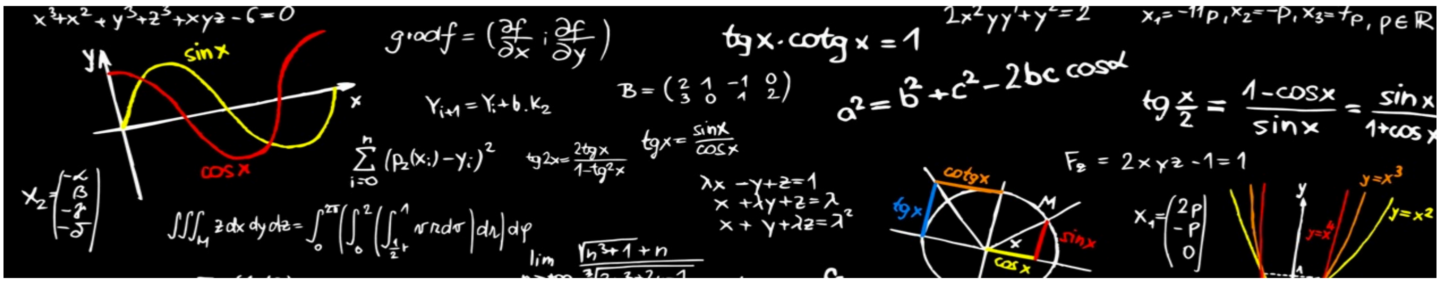

Abstract

An open problem for a long time was to find the set of all bounded Linear Functional of the Hardy Spaces in the complex disc D, usually denoted $H^1(D)$, which is the set of all analytic functions on the disc D, so that

$$\text{Sup}\int_0^{2\pi} |F(re^{i\theta})|d\theta < \infty \ over\ all\ r < 1.$$

In early 1970's, the young (and not well-known at the time) Charles Fefferman surprised the mathematical world by announcing that the dual of $H^1(D)$ (the set of all bounded linear functionals) was equivalent to the space of functions of bounded mean oscillation (BMO) on the boundary of the complex disc D. At around the same time, Ronald Coifman, announced an atomic decomposition of $H^1(D)$.

According to Yves Meyer, the late Antoni Zygmund commenting about the results of Fefferman and Coifman said: "The atomic decomposition of $H^1(D)$ is much more important than the duality of $H^1(D)$". Yves Meyer said: "I thought the Coifman result, was a different way to find the dual of $H^1(D)$. I was wrong."

The Atomic Decomposition of Coifman gave rise to De Souza's Special Atomic decomposition in 1978 and consequently the rebirth of Wavelets.

Finally, in 2010, De Souza discovered that the Lorentz Spaces L(p,1) for p>1, introduced in the early 1950's, also have a "Special Atomic Decomposition", justifying a famous theorem of Guido Weiss and Elias Stein that leads to an interpolation of operators.

Eddy Kwessi, Paul Alfonso, G de Souza and Ash Abebe in 2012 showed that, this new characterization of L(p,1) leads to a simple and straightforward way to study operators in L(p,q) (Lorentz Spaces), in particular L(p,p) = L_p, such as Multiplication, Composition, Maximal Operators, etc. as well as Carleson's maximal operator for Fourier series. The method also leads to a special atomic decomposition of the weighted and Lorentz-Bochner Spaces.