

Comprehensive Exam Syllabus (revised October 2014)

Disclaimer: *The topics listed below are intended to provide a guideline to the student preparing for the comprehensive examination in Analysis. It is by no means exhaustive.*

Real numbers -- Triangle inequality, Well ordering Principle, Induction, Supremum, Infimum, Completeness Axiom, Archimedean property, density of rational & irrational numbers.

Sequences -- convergence, boundedness, sub-sequences, limit theorems, squeeze theorem, monotone sequences, monotone convergence theorem, Bolzano-Weierstrass theorem, Cauchy sequences, limsup, liminf, Cauchy convergence criterion, completeness of reals.

Real Infinite series -- convergence, Cauchy criterion, absolute and conditional convergence, tests for convergence.

Continuity of real valued functions over reals -- Definition (using sequences, using epsilon/delta, using limit), Intermediate and Extreme value theorems, basic notions of limits, uniform continuity.

Derivative of a real valued function of one real variable -- Definition of derivative, operations of differentiable functions, Cauchy and Lagrange mean value theorems, Rolle's theorem, Taylor's theorem, Increasing/decreasing functions, 1-1 function, inverse of a function, derivative of the inverse.

Integration of a real valued function of one real variable -- Darboux sums, Riemann integrability, properties of integrals, First and Second Fundamental theorems of Calculus, Integral mean value theorem.

Power series -- convergence, radius and interval of convergence, Weierstrass M-test, term by term integration and differentiation of power series, Taylor series, power series for exponential, logarithmic and trigonometric functions, binomial series, etc.

Structure of Euclidean Space—Inner product, , Cauchy-Schwarz inequality, Triangle inequality, Open set definition, Open ball, closed set definition, interior, exterior and boundary of a set, limit point of a set.

Metric Spaces -- Definition of metric space, examples, Open & closed sets, convergent sequences, Cauchy sequences, Completeness, Compactness, Nested set property, Heine-Borel Theorem, Connectedness.

Continuous functions on a Metric space -- Definition, examples, continuity and limits, continuity of rational operations, functions in Euclidean space, uniform continuity, continuous functions on a compact metric space, continuous functions on a connected metric space.

Sequence of functions on a metric space -- point-wise and uniform convergence, space of continuous functions with sup norm metric, integration and differentiation of a sequence of real valued functions, interchange of limit operations.

Differentiability of functions of several real variables -- Limits in several variables, Partial derivatives, Mean Value Theorem in several variables, Directional derivatives, First and Second order approximation theorems, Positive definite quadratic functions, Hessian matrices, Second derivative test for local extrema of functions of several variables, Derivative matrix of a vector valued function of several variables, Chain rule, Inverse function theorem.

Suggested References

1. *Elementary Analysis: The theory of Calculus*, Kenneth A. Ross (Math 4130 material)
2. *Advanced Calculus*, P. M. Fitzpatrick (Math 4310 & Math 5320 materials)
3. *Elementary Classical Analysis*, J. Marsden and M. Hoffman, (Math 5320 material)
4. *Principles of Mathematical Analysis*, W. Rudin (Math 4310 & Math 5320 materials)