Department of Mathematics Spring 2002

Comprehensive Exam for Master's Degree - Analysis January 2002

1. Let

$$f_n(x) = \frac{1}{x^2 + n^2},\ x \in R,\ n \geq 1.$$

Prove or disprove: $f_n$ converges uniformly on $R$.

2. Let $f$ and $g$ be continuous real valued functions on $[0,1]$ with $f(0) = g(0) = 0$ and $f(1) = g(1) = 1$. Define the following space curve in $R^3$ :

$$\mathbf{r}(t) = (t, f(t), g(t)),\ t \in [0,1].$$

Show that $\mathbf{r}(t)$ must pass through the plane $x + y + z = \frac{3}{2}$ for some point $t_0 \in (0,1)$.

3. Let

$$G(x) = p_0 + p_1 x + p_2 x^2 + \cdots,$$

where

$$p_n \geq 0 \text{ for all } n \text{ and } \sum_{n=0}^{\infty} p_n = 1.$$

a. Show that $G(x)$ exists and is continuous on $[-1,1]$.

b. Conclude that $G(x)$ is infinitely differentiable on $(-1,1)$. State your reasons.

c. Suppose

$$p_n = \frac{1}{n+1} - \frac{1}{n+2}.$$

Does the limit of the derivative $G'(x)$ exist as $x$ tends to 1 from the left? State your reasoning.