Graph algebras: from analysis to algebra and back

There are a number of ways to associate an algebra to a graph, the most common one is the so called path algebra. Starting from this path algebra and taking suitable quotients, we arrive at deep matrices, Bergman algebras, Cohn path algebras, Leavitt path algebras and graph C*-algebras (if we add the structure of a C*-algebra).

Building on the pioneering work by Cuntz and Krieger in the 1980's, the study of graph C*-algebras took a rapid development. The investigation of Leavitt path algebras started more recently, around 2004. These can be considered as the algebraic analogue of graph C*-algebras; they also can be seen as a generalization of the algebras studied by Leavitt in a quest for algebras A without the property that all the bases of a given A-module have the same cardinality.

Many known algebras and C*-algebras can be described as the graph C*-algebra or as the Leavitt path algebra associated to a particular graph. Consequently, many structural properties can be expressed in terms of the underlying graph; examples are: simplicity, being simple and purely infinite, von Neumann regularity, stable rank, and others. This provides us with a convenient way of producing algebras and C*-algebras 'à la carte'.

There are deep connections between Leavitt path algebras, graph C*-algebras and other branches of Mathematics. We mention here the applications in K-theory and to Elliott's classification programme for certain separable nuclear C*-algebras. In our talk, we will introduce some of the ideas above and elaborate on this fascinating interplay between Analysis and Algebra.