Renormalization of frequency and scattering amplitude for water wave turbulence

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Abstract

In the classical theory of wave turbulence, the Hamiltonian for the weakly interacting gravity water waves is comprised of the energy of individual waves with wavenumbers k and frequencies ω_k and of the nonlinear term with matrix element λ_{1234} describing interactions of wave quartets in the otherwise quiet sea. In real turbulence, however, any wave propagates differently, which is reflected in the renormalization of the frequency of a single wave. In addition, the waves interact differently, which is reflected in the renormalization of the matrix element, or scattering amplitude, λ_{1234}. Recently, Rosenhaus and Falkovich have derived a new kinetic equation for four-wave interactions renormalized by multi-mode interactions in turbulence (arXiv:2308.00033). Using analytical methods from the quantum field theory, the authors arrive at conclusions that challenge classical Zakharov's theory of weak turbulence (Zakharov, Lvov, Falkovich, Kolmogorov spectra of turbulence, 1991) and open new horizons in wave-turbulence theory and applications of all kinds. For now, however, they managed to advance analysis to final expressions for the four-wave collision integral only in two toy models with simplified scattering amplitudes. These are generally more complex for interactions in real physical systems, and substantially more complex for surface water waves. In this talk, a numerical study of the first-order corrections for frequency and scattering amplitude for gravity water waves will be presented. We demonstrate that the first-order correction for frequency is positive and is linearly proportional to the wave number. The scattering amplitude and its correction exhibit a variety of features in the parameter space of four independent vector components. We focus on the case of resonant interactions and show that the interactions between parallel and anti-parallel waves are enhanced by turbulence.