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Title: A tensor-triangular approach to noncommutative geometry

Abstract: The prime spectrum of a commutative ring is the basic object of study in modern algebraic geometry. Balmer proved in the mid 2000s that there is a similar spectrum for a symmetric tensor category; but many tensor categories of interest, for instance arising from the representation theory of Hopf algebras, have non-symmetric (i.e. noncommutative) tensor products. We describe a recent approach, in part involving joint work with Daniel Nakano and Milen Yakimov, which associates to such a Hopf algebra (or, alternatively, to any finite tensor category) a geometric object, the noncommutative Balmer spectrum, which takes inspiration from the rich field of noncommutative ring theory. We discuss the major applications of Balmer spectra, including thick ideal classification problems, and approaches to concretely realizing Balmer spectra via support varieties based on cohomology rings.