Wandering in the Elastic Line

The elastic line (sometimes called the directed polymer) describes a 1 dimensional curve, living in some higher dimensional space, which is shaped by two competing forces: One force is the elasticity, which wants to minimize stretching, while the other force comes from the environment, which the line must navigate through to minimize obstructions. The original motivation for this model comes from Physics, where owing to its simplicity, it has been used to study the behavior of a huge number of systems. These include a polymer (such as DNA) in a solvent, the path of a mote of dust through an in-homogeneous fluid, and flux lines in superconductors.

However, mathematically, this model has drawn interest as it serves as one of the simplest models of a random curve which is non-diffusive, and it has in fact been shown to be one of the two "universal" examples in that class. Diffusivity, refers to whether the curve moves away from its origin at roughly the same rate as when one removes the environment. When it moves at the same speed, it is called diffusive, as opposed to super and sub diffusive when it moves faster or slower, respectively. A key prediction from the physics literature is that the behavior of the elastic line should radically change at a certain fixed temperature, where the model goes from being diffusive at high temperature, to being super diffusive at low temperature. This is an example of a phase transition. However, even this fundamental fact is only rigorously known in a few cases. In this talk, I will present work that shows that this question can be answered in the "high-dimensional limit", as well as survey what else is known, and what else is still open.

This talk is intended to be accessible to a wide audience, including graduate students and those with limited or no background in probability. Much of the talk will also be expository, and we will introduce the basic definitions and ideas behind how a physical system like this relates to a concrete and mathematical question.