Six wave interaction equations in finite-depth gravity waves with surface tension

Ziad Musslimani
Florida State University

Abstract

Three wave resonant triad interactions in two space and one time dimensions form a well-known system of first-order quadratically nonlinear evolution equations that arise in many areas of physics. In deep water waves, they were first derived by Simmons in 1969 and later shown to be exactly solvable by Ablowitz & Haberman in 1975. Specifically, integrability was established by introducing a system of six wave interactions whose symmetry reduction leads to the well-known three wave equations. Here, it is shown that the six wave interaction and classical three wave equations satisfying triad resonance conditions in finite-depth gravity waves can be derived from the non-local integro-differential formulation of the free surface gravity wave equation with surface tension. These quadratically nonlinear six wave interaction equations and their reductions to the classical and non-local complex as well as real reverse space–time three wave interaction equations are integrable. Limits to infinite and shallow water depth are also discussed.