FIXED POINTS AND CYCLES OF PARKING FUNCTIONS

Parking functions are combinatorial objects that lie between permutations and mappings. A parking function of length n is a sequence $\pi = (\pi_1, \dots, \pi_n)$ of positive integers such that if $\lambda_1 \leq \cdots \leq \lambda_n$ is the increasing rearrangement of $\pi_1, \dots, \pi_n$, then $\lambda_i \leq i$ for $1 \leq i \leq n$. The index i is a fixed point of the parking function π if $\pi_i = i$. More generally, for $m \geq 1$, the indices $(i_1, \dots, i_m)$ where the i_j's are all distinct constitute an m-cycle of the parking function π if $\pi_{i_1} = i_2, \pi_{i_2} = i_3, \dots, \pi_{i_{m-1}} = i_m, \pi_{i_m} = i_1$.