

# The Generalized Riemann Problem for the Korteweg–De Vries Equation

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## Abstract

The Riemann problem, considering an initial configuration of a discontinuous, step-like jump between two constant values, is a fundamental problem in dispersive hydrodynamics. This work extends the Riemann problem to address interactions between two periodic traveling waves, termed the Generalized Riemann problem, for the Korteweg–De Vries equation. The examination starts with special classes of two-phase wave interactions consisting of two cnoidal wave solutions subject to a rapid transition in one wave parameter. Numerical simulations are performed for six distinct initial data cases, yielding the long-term emergence of two-phase *bright/dark breather dispersive shock waves (DSWs)* characterized by a bright/dark breather train edge and a harmonic edge, as well as one-phase rarefaction wave-like structures with internal oscillations. The characteristic velocities of the waves are compared with predictions from multiphase Whitham modulation theory.