

## Application of Regular Local Rings to Number Theory

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Let  $D$  be an  $n$ -dimensional regular local ring. A quadratic transform of  $D$  is obtained by adding some carefully chosen fractions and localizing. Such a quadratic transform of  $D$  - of which there are infinitely many - is again an  $n$ -dimensional regular local ring. Hence, this process can be iterated, giving rise to a quadratic transform tree. In 1956 Abhyankar proved some marvelous properties of this tree in the case where  $n = 2$ . In the early 1970s David Shannon (an Abhyankar student) was assigned the problem of determining to what extent the nice two-dimensional properties of this tree extend to higher dimensions. Answer: Almost everything fails in three dimensions. But the failure is not uniform. Rather there can be a mix of failure and success; some branches of the tree give us the results we want and some do not. We analyzed this mix and discovered some fascinating connections to number theory. Of particular interest is a notion of higher dimensional continued fractions.