

On the non-holonomic character of first time returns in the Euclidean lattice

We take a combinatorial approach to the classical problem of random walks in the d -dimensional Euclidean lattice \mathbb{Z}^d , where the walker takes steps $\{-1, +1\}^d$, each with equal probability $1/(2d)$. We give precise asymptotics for the number of closed walks or excursions (the walks of a given length that return to the starting point). We also discuss the more complicated case of first-time returns.

Recall that a sequence $(f_n)_n$ of integers is called P -recursive if it admits a recurrence relation with coefficients polynomials in n . This property is equivalent with the generating function being holonomic (or D -finite), meaning that the generating function of $(f_n)_n$ satisfies an ordinary differential equation with polynomial coefficients. We show that, in dimension greater or equal to 2, the sequence of excursions is P -recursive, while the sequence of first-time returns is not P -recursive. Techniques from the theory of Legendre polynomials, Hadamard convolution, analytic combinatorics, and Fuchsian ODEs are employed in the proofs.

This presentation reports on joint work with Liviu Suciu. The first part is accessible to undergraduate students.