Rings & Wings

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Title: The Lord of the Moves

Abstract: Combinatorics of graphs is a very powerful tool to unravel various properties of graph algebras. In particular, isomorphisms between graph algebras are often implemented by moves between their graphs. In this talk, I will review the landscape of Middle-earth of graph moves to show how to make these combinatorial methods functorial. Our categorical approach not only reveals the relationship between moves, but also leads to unexpected extensions of known moves (shifts and reductions) and the characterization of out-splits as admissible epimorphisms admitting a lifting of the source map. The former allows us to go beyond isomorphisms and Morita equivalences, and the latter yields a factorization lemma for out-splits. Better still, the functorial understanding of maximal out-splits permits combining them with the derived-graph construction, which results in computing new proper endomorphisms of Cuntz algebras. Finally, the functorial implementation of balanced in-splits requires the construction of a new category of graphs, where morphisms are given as homomorphisms of graph inverse semigroups. Thus, the combinatorics of moves brings a new tool to the realm of noncommutative topology where directed graphs are viewed as quantum spaces. Time permitting, I will end the talk discussing our new quest to find one move to rule them all. (Based on joint work with G. G. de Castro, M. Lowiel, E. A. Pacheco, and J. R. R. Tolkien.)