

Modulation theory of Y-wave Kadomtsev-Petviashvili II Solitons

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The evolution of long wavelength perturbations of traveling wave 2-soliton solutions of the Kadomtsev-Petviashvili II (KP) equation are investigated using modulation theory. An initial-boundary value problem for a system of hyperbolic conservation laws is formulated for amplitude and slope perturbations to soliton legs of resonant (Y-shaped) KP traveling wave solutions. The vertex where individuated soliton legs coalesce is modeled as a moving free boundary where information between the soliton legs can pass. The linearized hyperbolic system is proven to be well-posed under suitable initial conditions and is solved exactly. All admissible simple (rarefaction) waves of the nonlinear system are characterized. Specific perturbations of Y-waves can lead to the dimensionality of the modulated solution changing. Spontaneous leg deletion (the appearance of vacuum) can occur in select singular and asymptotically singular cases, while leg creation can occur through gradient catastrophe after passing the vertex. The conditions under which the Y soliton modulation system can shock are given. The shock is regularized through Rankine-Hugoniot conditions, with an augmented system proposed to describe the growth of the new leg. The theoretical modulation predictions are supported by direct numerical simulations of KP II, which show good qualitative agreement throughout and, in most cases, good quantitative agreement as well.