Modulation Theory and Dispersive Shock Waves in a Regularized Boussinesq Equation

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Modulation theory for periodic traveling wave solutions of a quasi-continuum model of the Fermi-Pasta-Ulam-Tsingou lattice equation is developed. The Whitham modulation equations for a regularized Boussinesq equation are derived and their hyperbolicity investigated analytically and numerically for quadratic and cubic nonlinearity. Within regions of strict hyperbolicity, integral curves are computed that reconstruct dispersive shock wave (DSW) solutions resulting from the self-similar, long time dynamics of step-like initial data. The obtained integral curves are used as a benchmark to assess the accuracy of the DSW fitting method and a recently proposed alternative approach involving the solitary wave limit of the Whitham equations.