Rings & Wings

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Title: The Graded Naimark's Problem for Leavitt Path Algebras

Abstract: This is a joint work with Ashish Srivastava. In 1951, Naimark showed that the C^*-algebra K of compact operators on any Hilbert space has exactly one irreducible representation up to unitary equivalence, and asked whether a C^*-algebra satisfying this property is isomorphic to K. After more than 50 years of attempts by various researchers, Akemann and Weaver showed in 2004 that an answer to Naimark's question for arbitrary C^*-algebras is undecidable under ZFC. Recently Mark Tomforde and I considered the case of graph C^*-algebras, and using graphical techniques we were able to show that Naimark's problem has an affirmative answer for graph C^*-algebras $C^*(E)$ over an arbitrary graph E. We were also able to show that the same graphical conditions apply to answer the algebraic version of Naimark's problem for Leavitt path algebras $L_K(E)$ of a graph E over a field K. Now Leavitt path algebras are $\mathbb{Z}$-graded algebras. In this talk we consider the graded version of Naimark's problem. After noting that, in the context of a graded algebra, the unitary equivalence of irreducible representations is the same thing as graded isomorphism of graded simple modules, we completely characterize those Leavitt path algebras $L_K(E)$ having exactly one graded simple module up to graded isomorphism. In this case, the graph E is shown to be row-finite, downward directed, and E^0 is the hereditary saturated closure of a single vertex v which is either a line point, or lies on a cycle without exits in E. We also characterize Leavitt path algebras possessing finite or countably infinite graded isomorphism classes of graded simple left/right modules.