The ARCS Seminar

Universal localization at semiprime Goldie ideals

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Abstract: For a commutative Noetherian ring R with prime ideal P, the localization R_P of R with respect to P is the ring universal with respect to inverting the elements in the complement of P. For a noncommutative Noetherian ring R, after replacing the complement of P with the set of elements $C(P)$ that are not zero divisors modulo P, the same construction can be made, but it does not appear to yield anything useful.

P. M. Cohn (in 1973) replaced the elements $C(P)$ with the set $\Gamma(P)$ of matrices that are not zero divisors modulo P, and showed that the resulting localization $R_{\Gamma(P)}$ has the desirable property that modulo its Jacobson radical it is isomorphic to the classical ring of quotients of R/P. I showed (in 1981) that $R_{\Gamma(P)}$ can be defined as the ring universal with respect to this property. (This is also a generalization of what happens in the commutative case, though that definition of R_P is apparently not be as well known as the definition given via inverting elements.)

I would note that Cohn's localization is related to one defined earlier by Goldie (in 1967). It behaves reasonably modulo powers of its Jacobson radical, but the kernel of the canonical mapping from R to $R_{\Gamma(P)}$ is very difficult to compute in most cases. It also seems to be on the opposite end of some spectrum involving the torsion theoretic localization, which has been extensively studied, and which is basically defined from the bottom up rather than from the top down.

The advantage of using a definition that depends on a universal property is that it probably always exists. The disadvantage is that it may be very difficult to compute, and that is certainly the case for $R_{\Gamma(P)}$. But, like an injective envelope or algebraic closure, it may still play a useful role in certain circumstances.

I will state some of the basic properties of $R_{\Gamma(P)}$, give two different constructions, and attempt to show how $R_{\Gamma(P)}$ can be used to provide a language that exploits similarities to the commutative case.

Time and Place: Wednesday, April 3 from 4:30–5:30PM (Mountain Time Zone) in ENG 187

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