



## The ARCS Seminar

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### Modules in Which Products Commute

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**Abstract:** The definition of a product  $L * N$  for submodules  $L, N$  of a module  $M$  was defined almost 50 years ago, but I can't find any reference asking when  $L * N = N * L$  for all submodules.

The definition:  $L * N$  is the sum of all homomorphic images  $f(N)$  under homomorphisms  $f : M \rightarrow N$ . I will explain why this seems like an appropriate generalization of the product of two left ideals.

Any cyclic module over a commutative ring satisfies the condition, because it is a commutative ring in its own right and the multiplication of submodules is ordinary multiplication. But over a principal ideal domain, for example, the cyclic modules are the only finitely generated modules that satisfy the commutativity condition.

I have a few results about modules over commutative rings, and about modules over Artinian rings, but everything is at a very preliminary stage.

**Time and Place:** Wednesday, Nov. 8 from 3:30–4:30PM (Mountain Time Zone) in ENG 187



The Rings and Wings Seminar is an activity of ARCS.

<https://arcs-center.org>