The ARCS Seminar

Sandpile models and weighted Leavitt path algebras

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Abstract: The notion of sandpile models encapsulates a process by which objects may spread and evolve along a grid. The models were conceived in 1987 in the seminal paper by Bak, Tang and Wiesenfeld as an example of *self-organized criticality*, or the tendency of physical systems to organise themselves without any input from outside the system, toward critical but barely stable states. The models were used to describe phenomena such as forest fires, traffic jams, stock market fluctuations, etc. In subsequent major work (1990), Dhar championed the use of an abelian group naturally associated to a sandpile model as an invariant which was shown to capture many properties of the model. This abelian group is paired with a naturally-arising monoid that arises from the grid.

In a different realm, the notion of Leavitt path algebras $L_K(E)$ associated to directed graphs E, with coefficients in a field K, were introduced in 2005. These are a generalisation of algebras (denoted by $L_K(1, 1+k)$) introduced by William Leavitt in 1962; these "Leavitt algebras" arise as the universal ring of type $(1, 1+k)$ (i.e., $A \cong A^{1+k}$ as right A-modules, where $k \in \mathbb{N}^+$). In fact Leavitt established much more in the 1962 article: he showed that for any $n, k \in \mathbb{N}$ that there is a universal ring A of type $(n, n+k)$ (denoted $L_K(n, n+k)$) for which $A^n \cong A^{n+k}$ as right A-modules. When $n \geq 2$, this universal ring is *not* realizable as a Leavitt path algebra. With this in mind, the notion of *weighted* Leavitt path algebras $L_K(E, w)$ associated to weighted graphs (E, w) were introduced by Hazrat in 2011. The weighted Leavitt path algebras $L_K(E, w)$ provide a natural (but *extremely broad*) context in which all of Leavitt's algebras (corresponding to any pair $n, k \in \mathbb{N}$) can be realised as a specific example.

The study of the commutative monoid $\mathcal{V}(B)$ of isomorphism classes of finitely generated projective right modules of a unital ring B (with operation $\oplus$) goes back to the work of Grothendieck and Serre. For a Leavitt path algebra $L_{\mathsf{k}}(E)$, the monoid $\mathcal{V}(L_K(E))$ has received substantial attention since the introduction of the topic. Furthermore, the monoid $\mathcal{V}(L_K(E, w))$ has been described in work by Hazrat, and subsequently by Preusser.

In this talk we'll show how the notions of sandpile monoids and weighted Leavitt path algebras are quite naturally related, via the $\mathcal{V}$-monoid. This relationship allows us to associate an algebra, a *sandpile algebra*, to the theory of sandpile models, thereby opening up an avenue by which to investigate sandpile models via the structure of the sandpile algebras, and vice versa. The sandpile algebras provide a natural (but *significantly more focused*) context in which all of Leavitt's algebras can be realized as a specific example.

This is joint work with Roozbeh Hazrat, Western Sydney University.

Time and Place: Wednesday, February 23 from 4:30–5:30PM (Mountain Time Zone) in ENGR 187

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