Symbolic computation of solitary wave solutions and solitons through homogenization of degree

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Abstract

A simplified version of Hirota's method for the computation of solitary waves and solitons of nonlinear PDEs will be presented. The approach requires a change of dependent variable so that the transformed PDE is homogenous of degree in that new variable. The resulting homogenous PDE does not have to be quadratic and the method still applies if its bilinear form is not known. Solitons are then computed using a perturbation scheme involving linear and nonlinear operators. For soliton equations the scheme terminates after a finite number of steps.

To illustrate the approach, solitons are computed for a class of fifth-order KdV equations due to Lax, Sawada-Kotera, and Kaup-Kupershmidt. Homogenization of degree also allows one to find solitary wave solutions of nonlinear PDEs that are not completely integrable. Examples include the Fisher and FitzHugh-Nagumo equations, and a combined KdV-Burgers equation. When applied to a wave equation with a cubic source term, the method leads to a 'bi-soliton' solution which describes the coalescence of two wavefronts.

The method is largely algorithmic and currently being implemented in Mathematica.