Generalized Pseudospectral Method and Zeros of Orthogonal Polynomials

Consider a polynomial family $\{p_\nu(x)\}_{\nu=0}^{\infty}$ orthogonal with respect to a measure supported on the real line, for which each polynomial in the family satisfies the differential equation $\mathscr{A} p_\nu(x) = q_\nu(x) p_\nu(x)$, where $\mathscr{A}$ is a linear differential operator and each $q_\nu(x)$ is a polynomial of degree at most $n_0 \in \mathbb{N}$, which does not depend on ν. Because, in general, the differential operator $\mathscr{A}$ raises the degree of polynomials by a summand of n_0, the standard pseudospectral method does not allow exact discretization of such differential equations. A generalization of the pseudospectral method is presented which allows matrix representations of a differential operator to depend on two rather that one set of interpolation nodes, thus yielding exact discretizations as desired. Via this generalization, a family of nonlinear algebraic identities satisfied by the zeros of a wide class of orthogonal polynomials is derived. Here we present the general case of these identities, and illustrate their application to the zeros of the Sonin-Markov polynomials, in which we also include the generalized pseudospectral representations of the differential operator $\mathscr{A}$ for the case of these polynomials.